9

Differential Equations

Short Answer Type Questions

Q. 1 Find the solution of $\frac{dy}{dx} = 2^{y-x}$.

Sol. Given that, $$\frac{dy}{dx} = 2^{y-x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2^y}{2^x} \qquad \left[\because a^{m-n} = \frac{a^m}{a^n}\right]$$

$$\Rightarrow \quad \frac{dy}{2^y} = \frac{dx}{2^x}$$

On integrationg both sides, we get

$$\int 2^{-y} dy = \int 2^{-x} dx$$

$$\Rightarrow \quad \frac{-2^{-y}}{\log 2} = \frac{-2^{-x}}{\log 2} + C$$

$$\Rightarrow \quad -2^{-y} + 2^{-x} = + C \log 2$$

$$\Rightarrow \quad 2^{-x} - 2^{-y} = - C \log 2$$

$$\Rightarrow \quad 2^{-x} - 2^{-y} = K \qquad [\text{where, } K = + C \log 2]$$

Q. 2 Find the differential equation of all non-vertical lines in a plane.

Sol. Since, the family of all non-vertical line is $y = mx + c$, where $m \neq \tan \frac{\pi}{2}$.

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = m$$

Again, differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = 0$$

Q. 3 If $\frac{dy}{dx} = e^{-2y}$ and $y = 0$ when $x = 5$, then find the value of x when $y = 3$.

Sol. Given that, $\frac{dy}{dx} = e^{-2y} \Rightarrow \frac{dy}{e^{-2y}} = dx$

$$\Rightarrow \quad \int e^{2y}\, dy = \int dx \Rightarrow \frac{e^{2y}}{2} = x + C \qquad \ldots \text{(i)}$$

When $x = 5$ and $y = 0$, then substituting these values in Eq. (i), we get

$$\frac{e^0}{2} = 5 + C$$

$$\Rightarrow \quad \frac{1}{2} = 5 + C \quad \Rightarrow \quad C = \frac{1}{2} - 5 = -\frac{9}{2}$$

Eq. (i) becomes $e^{2y} = 2x - 9$

When $y = 3$, then $e^6 = 2x - 9 \Rightarrow 2x = e^6 + 9$

$$\therefore \quad x = \frac{(e^6 + 9)}{2}$$

Q. 4 Solve $(x^2 - 1)\dfrac{dy}{dx} + 2xy = \dfrac{1}{x^2 - 1}$.

Sol. Given differential equation is

$$(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{1}{x^2 - 1}$$

$$\Rightarrow \quad \frac{dy}{dx} + \left(\frac{2x}{x^2 - 1}\right) y = \frac{1}{(x^2 - 1)^2}$$

which is a linear differential equation.

On comparing it with $\dfrac{dy}{dx} + Py = Q$, we get

$$P = \frac{2x}{x^2 - 1}, \; Q = \frac{1}{(x^2 - 1)^2}$$

$$\text{IF} = e^{\int P dx} = e^{\int \left(\frac{2x}{x^2 - 1}\right) dx}$$

Put $x^2 - 1 = t \Rightarrow 2x dx = dt$

$$\therefore \quad \text{IF} = e^{\int \frac{dt}{t}} = e^{\log t} = t = (x^2 - 1)$$

The complete solution is

$$y \cdot \text{IF} = \int Q \cdot \text{IF} + K$$

$$\Rightarrow \quad y \cdot (x^2 - 1) = \int \frac{1}{(x^2 - 1)^2} \cdot (x^2 - 1)\, dx + K$$

$$\Rightarrow \quad y \cdot (x^2 - 1) = \int \frac{dx}{(x^2 - 1)} + K$$

$$\Rightarrow \quad y \cdot (x^2 - 1) = \frac{1}{2} \log\left(\frac{x - 1}{x + 1}\right) + K$$

Q. 5 Solve $\dfrac{dy}{dx} + 2xy = y$.

Sol. Given that, $\dfrac{dy}{dx} + 2xy = y$

$$\Rightarrow \quad \frac{dy}{dx} + 2xy - y = 0$$

$$\Rightarrow \quad \frac{dy}{dx} + (2x - 1)\, y = 0$$

which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = (2x - 1), Q = 0$$

$$\text{IF} = e^{\int P dx} = e^{\int (2x-1)\,dx}$$

$$= e^{\left(\frac{2x^2}{2} - x\right)} = e^{x^2 - x}$$

The complete solution is

$$y \cdot e^{x^2 - x} = \int Q \cdot e^{x^2 - x}\,dx + C$$

$\Rightarrow$ $$y \cdot e^{x^2 - x} = 0 + C$$

$\Rightarrow$ $$y = C\,e^{x - x^2}$$

Q. 6 Find the general solution of $\frac{dy}{dx} + ay = e^{mx}$.

Sol. Given differential equation is

$$\frac{dy}{dx} + ay = e^{mx}$$

which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = a, Q = e^{mx}$$

$$\text{IF} = e^{\int P dx} = e^{\int a dx} = e^{ax}$$

The general solution is $$y \cdot e^{ax} = \int e^{mx} \cdot e^{ax}\,dx + C$$

$\Rightarrow$ $$y \cdot e^{ax} = \int e^{(m+a)x}\,dx + C$$

$\Rightarrow$ $$y \cdot e^{ax} = \frac{e^{(m+a)x}}{(m+a)} + C$$

$\Rightarrow$ $$(m+a)\,y = \frac{e^{(m+a)x}}{e^{ax}} + \frac{(m+a)C}{e^{ax}}$$

$\Rightarrow$ $$(m+a)\,y = e^{mx} + K\,e^{-ax}$$ $[\because K = (m+a)C]$

Q. 7 Solve the differential equation $\frac{dy}{dx} + 1 = e^{x+y}$.

Sol. Given differential equation is $$\frac{dy}{dx} + 1 = e^{x+y} \quad \text{... (i)}$$

On substituting $x + y = t$, we get

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

Eq. (i) becomes $$\frac{dt}{dx} = e^t$$

$\Rightarrow$ $$e^{-t} dt = dx$$

$\Rightarrow$ $$-e^{-t} = x + C$$

$\Rightarrow$ $$\frac{-1}{e^{x+y}} = x + C$$

$\Rightarrow$ $$-1 = (x + C)\,e^{x+y}$$

$\Rightarrow$ $$(x + C)\,e^{x+y} + 1 = 0$$

Q. 8 Solve $ydx - xdy = x^2 ydx$.

Sol. Given that,
$$ydx - xdy = x^2 ydx$$

$$\Rightarrow \quad \frac{1}{x^2} - \frac{1}{xy} \cdot \frac{dy}{dx} = 1 \quad \text{[dividing throughout by } x^2 ydx]$$

$$\Rightarrow \quad -\frac{1}{xy} \cdot \frac{dy}{dx} + \frac{1}{x^2} - 1 = 0$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{xy}{x^2} + xy = 0$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} + xy = 0$$

$$\Rightarrow \quad \frac{dy}{dx} + \left(x - \frac{1}{x}\right) y = 0$$

which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = \left(x - \frac{1}{x}\right), Q = 0$$

$$\text{IF} = e^{\int P dx}$$

$$= e^{\int \left(x - \frac{1}{x}\right) dx}$$

$$= e^{\frac{x^2}{2} - \log x}$$

$$= e^{\frac{x^2}{x}}, e^{-\log x}$$

$$= \frac{1}{x} e^{\frac{x^2}{2}}$$

The general solution is

$$y \cdot \frac{1}{x} e^{x^2/2} = \int 0 \cdot \frac{1}{x} e^{x^2/2}\, dx + C$$

$$\Rightarrow \quad y \cdot \frac{1}{x} e^{x^2/2} = C$$

$$\Rightarrow \quad y = C\, x\, e^{-x^2/2}$$

Q. 9 Solve the differential equation $\frac{dy}{dx} = 1 + x + y^2 + xy^2$, when $y = 0$ and $x = 0$.

Sol. Given that,
$$\frac{dy}{dx} = 1 + x + y^2 + xy^2$$

$$\Rightarrow \quad \frac{dy}{dx} = (1 + x) + y^2 (1 + x)$$

$$\Rightarrow \quad \frac{dy}{dx} = (1 + y^2)(1 + x)$$

$$\Rightarrow \quad \frac{dy}{1 + y^2} = (1 + x)\, dx$$

On integrating both sides, we get

$$\tan^{-1} y = x + \frac{x^2}{2} + K \quad \text{... (i)}$$

When $y = 0$ and $x = 0$, then substituting these values in Eq. (i), we get

$$\tan^{-1}(0) = 0 + 0 + K$$

$$\Rightarrow \quad K = 0$$

$$\Rightarrow \quad \tan^{-1} y = x + \frac{x^2}{2}$$

$$\Rightarrow \quad y = \tan\left(x + \frac{x^2}{2}\right)$$

Q. 10 Find the general solution of $(x + 2y^3)\dfrac{dy}{dx} = y$.

Sol. Given that,

$$(x + 2y^3)\frac{dy}{dx} = y$$

$$\Rightarrow \quad y \cdot \frac{dx}{dy} = x + 2y^3$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{x}{y} + 2y^2 \qquad \text{[dividing throughout by } y]$$

$$\Rightarrow \quad \frac{dx}{dy} - \frac{x}{y} = 2y^2$$

which is a linear differential equation.

On comparing it with $\dfrac{dx}{dy} + Px = Q$, we get

$$P = -\frac{1}{y}, Q = 2y^2$$

$$\text{IF} = e^{\int -\frac{1}{y} dy} = e^{-\int \frac{1}{y} dy}$$

$$\therefore \quad = e^{-\log y} = \frac{1}{y}$$

The general solution is

$$x \cdot \frac{1}{y} = \int 2y^2 \cdot \frac{1}{y} dy + C$$

$$\Rightarrow \quad \frac{x}{y} = \frac{2y^2}{2} + C$$

$$\Rightarrow \quad \frac{x}{y} = y^2 + C$$

$$\Rightarrow \quad x = y^3 + Cy$$

Q. 11 If $y(x)$ is a solution of $\left(\dfrac{2 + \sin x}{1 + y}\right)\dfrac{dy}{dx} = -\cos x$ and $y(0) = 1$, then find the value of $y\left(\dfrac{\pi}{2}\right)$.

Sol. Given that,

$$\left(\frac{2 + \sin x}{1 + y}\right)\frac{dy}{dx} = -\cos x$$

$$\Rightarrow \quad \frac{dy}{1 + y} = -\frac{\cos x}{2 + \sin x} dx$$

On integrating both sides, we get

$$\int \frac{1}{1 + y} dy = -\int \frac{\cos x}{2 + \sin x} dx$$

$$\Rightarrow \quad \log(1 + y) = -\log(2 + \sin x) + \log C$$

$$\Rightarrow \quad \log(1+y) + \log(2+\sin x) = \log C$$
$$\Rightarrow \quad \log(1+y)(2+\sin x) = \log C$$
$$\Rightarrow \quad (1+y)(2+\sin x) = C$$
$$\Rightarrow \quad 1+y = \frac{C}{2+\sin x}$$
$$\Rightarrow \quad y = \frac{C}{2+\sin x} - 1 \quad \text{... (i)}$$

When $x = 0$ and $y = 1$, then

$$1 = \frac{C}{2} - 1$$
$$\Rightarrow \quad C = 4$$

On putting $C = 4$ in Eq. (i), we get

$$y = \frac{4}{2+\sin x} - 1$$
$$\therefore \quad y\left(\frac{\pi}{2}\right) = \frac{4}{2+\sin\frac{\pi}{2}} - 1 = \frac{4}{2+1} - 1$$
$$= \frac{4}{3} - 1 = \frac{1}{3}$$

Q. 12 If $y(t)$ is a solution of $(1+t)\dfrac{dy}{dt} - ty = 1$ and $y(0) = -1$, then show that $y(1) = -\dfrac{1}{2}$.

Sol. Given that, $$(1+t)\frac{dy}{dt} - ty = 1$$
$$\Rightarrow \quad \frac{dy}{dt} - \left(\frac{t}{1+t}\right)y = \frac{1}{1+t}$$

which is a linear differential equation.

On comparing it with $\dfrac{dy}{dt} + Py = Q$, we get

$$P = -\left(\frac{t}{1+t}\right), Q = \frac{1}{1+t}$$
$$\text{IF} = e^{-\int \frac{t}{1+t}dt} = e^{-\int\left(1-\frac{1}{1+t}\right)dt} = e^{-[t-\log(1+t)]}$$
$$= e^{-t} \cdot e^{\log(1+t)}$$
$$= e^{-t}(1+t)$$

The general solution is

$$y(t) \cdot \frac{(1+t)}{e^t} = \int \frac{(1+t) \cdot e^{-t}}{(1+t)} dt + C$$
$$\Rightarrow \quad y(t) = \frac{e^{-t}}{(-1)} \cdot \frac{e^t}{1+t} + C', \text{ where } C' = \frac{C e^t}{1+t}$$
$$\Rightarrow \quad y(t) = -\frac{1}{1+t} + C'$$

When $t = 0$ and $y = -1$, then

$$-1 = -1 + C' \Rightarrow C' = 0$$
$$y(t) = -\frac{1}{1+t} \Rightarrow y(1) = -\frac{1}{2}$$

Q. 13 **Form the differential equation having $y = (\sin^{-1} x)^2 + A\cos^{-1} x + B$, where A and B are arbitrary constants, as its general solution.**

Sol. Given that, $y = (\sin^{-1} x)^2 + A\cos^{-1} x + B$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = \frac{2\sin^{-1} x}{\sqrt{1-x^2}} + \frac{(-A)}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad \sqrt{1-x^2}\,\frac{dy}{dx} = 2\sin^{-1} x - A$$

Again, differentiating w.r.t. x, we get

$$\sqrt{1-x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{-2x}{2\sqrt{1+x^2}} = \frac{2}{\sqrt{1-x^2}}$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}}\cdot\sqrt{1-x^2}\,\frac{dy}{dx} = 2$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$$

$$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$$

which is the required differential equation.

Q. 14 **Form the differential equation of all circles which pass through origin and whose centres lie on Y-axis.**

Sol. It is given that, circles pass through origin and their centreslie on Y-axis. Let $(0, k)$ be the centre of the circle and radius is k.

So, the equation of circle is

$$(x-0)^2 + (y-k)^2 = k^2$$

$$\Rightarrow \quad x^2 + (y-k)^2 = k^2$$

$$\Rightarrow \quad x^2 + y^2 - 2ky = 0$$

$$\Rightarrow \quad \frac{x^2+y^2}{2y} = k \qquad \text{... (i)}$$

On differentiating Eq. (i) w.r.t. x, we get

$$\frac{2y\left(2x + 2y\frac{dy}{dx}\right) - (x^2+y^2)\frac{2dy}{dx}}{4y^2} = 0$$

$$\Rightarrow \quad 4y\left(x + y\frac{dy}{dx}\right) - 2(x^2+y^2)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad 4xy + 4y^2\frac{dy}{dx} - 2(x^2+y^2)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad [4y^2 - 2(x^2+y^2)]\frac{dy}{dx} + 4xy = 0$$

$$\Rightarrow \quad (4y^2 - 2x^2 - 2y^2)\frac{dy}{dx} + 4xy = 0$$

$$\Rightarrow \quad (2y^2 - 2x^2)\frac{dy}{dx} + 4xy = 0$$

$$\Rightarrow \quad (y^2 - x^2)\frac{dy}{dx} + 2xy = 0$$

$$\Rightarrow \quad (x^2 - y^2)\frac{dy}{dx} - 2xy = 0$$

Q. 15 Find the equation of a curve passing through origin and satisfying the differential equation $(1+x^2)\dfrac{dy}{dx}+2xy=4x^2$.

Sol. Given that,
$$(1+x^2)\frac{dy}{dx}+2xy=4x^2$$
$$\Rightarrow \quad \frac{dy}{dx}+\frac{2x}{1+x^2}\cdot y=\frac{4x^2}{1+x^2}$$

which is a linear differential equation.

On comparing it with $\dfrac{dy}{dx}+Py=Q$, we get
$$P=\frac{2x}{1+x^2}, Q=\frac{4x^2}{1+x^2}$$
$$\therefore \quad \text{IF}=e^{\int P dx}=e^{\int \frac{2x}{1+x^2}dx}$$

Put $1+x^2=t \Rightarrow \quad 2x dx=dt$
$$\text{IF}=1+x^2=e^{\int \frac{dt}{t}}=e^{\log t}=e^{\log(1+x^2)}$$

The general solution is
$$y\cdot(1+x^2)=\int \frac{4x^2}{1+x^2}(1+x^2)\,dx+C$$
$$\Rightarrow \quad y\cdot(1+x^2)=\int 4x^2\,dx+C$$
$$\Rightarrow \quad y\cdot(1+x^2)=4\frac{x^3}{3}+C \qquad \text{... (i)}$$

Since, the curve passes through origin, then substituting

$x=0$ and $y=0$ in Eq. (i), we get

$C=0$

The required equation of curve is
$$y(1+x^2)=\frac{4x^3}{3}$$
$$\Rightarrow \quad y=\frac{4x^3}{3(1+x^2)}$$

Q. 16 Solve $x^2\dfrac{dy}{dx}=x^2+xy+y^2$.

Sol. Given that,
$$x^2\frac{dy}{dx}=x^2+xy+y^2$$
$$\Rightarrow \quad \frac{dy}{dx}=1+\frac{y}{x}+\frac{y^2}{x^2} \qquad \text{... (i)}$$

Let
$$f(x,y)=1+\frac{y}{x}+\frac{y^2}{x^2}$$
$$f(\lambda x,\lambda y)=1+\frac{\lambda y}{\lambda x}+\frac{\lambda^2 y^2}{\lambda^2 x^2}$$
$$f(\lambda x,\lambda y)=\lambda^0\left(1+\frac{y}{x}+\frac{y^2}{x^2}\right)$$
$$=\lambda^0 f(x,y)$$

which is homogeneous expression of degree 0.

Put $$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

On substituting these values in Eq.(i), we get

$$\left(v + x\frac{dv}{dx}\right) = 1 + v + v^2$$

$$\Rightarrow \quad x\frac{dv}{dx} = 1 + v + v^2 - v$$

$$\Rightarrow \quad x\frac{dv}{dx} = 1 + v^2$$

$$\Rightarrow \quad \frac{dv}{1+v^2} = \frac{dx}{x}$$

On integrating both sides, we get

$$\tan^{-1} v = \log|x| + C$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{y}{x}\right) = \log|x| + C$$

Q. 17 Find the general solution of the differential equation $(1 + y^2) + (x - e^{\tan^{-1} y})\dfrac{dy}{dx} = 0.$

Sol. Given, differential equation is

$$(1 + y^2) + (x - e^{\tan^{-1} y})\frac{dy}{dx} = 0$$

$$\Rightarrow \quad (1 + y^2) = -(x - e^{\tan^{-1} y})\frac{dy}{dx}$$

$$(1 + y^2)\frac{dx}{dy} = -x + e^{\tan^{-1} y}$$

$$\Rightarrow \quad (1 + y^2)\frac{dx}{dy} + x = e^{\tan^{-1} y}$$

$$\Rightarrow \quad \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1} y}}{1+y^2} \quad \text{[dividing throughout by } (1+y^2)]$$

which is a linear differential equation.

On comparing it with $\dfrac{dx}{dy} + Px = Q$, we get

$$P = \frac{1}{1+y^2}, \; Q = \frac{e^{\tan^{-1} y}}{1+y^2}$$

$$\text{IF} = e^{\int P dy} = e^{\int \frac{1}{1+y^2} dy} = e^{\tan^{-1} y}$$

The general solution is $$x \cdot e^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1+y^2} \cdot e^{\tan^{-1} y}\, dy + C$$

$$\Rightarrow \quad x \cdot e^{\tan^{-1} y} = \int \frac{(e^{\tan^{-1} y})^2}{1+y^2} \cdot dy + C$$

Put $\tan^{-1} y = t \Rightarrow$ $$\frac{1}{1+y^2}dy = dt$$

$$\therefore \quad x \cdot e^{\tan^{-1} y} = \int e^{2t}\, dt + C$$

$$\Rightarrow \quad x \cdot e^{\tan^{-1} y} = \frac{1}{2} e^{2\tan^{-1} y} + C$$

$$\Rightarrow \quad 2x\, e^{\tan^{-1} y} = e^{2\tan^{-1} y} + 2C$$

$$\Rightarrow \quad 2x\, e^{\tan^{-1} y} = e^{2\tan^{-1} y} + K \qquad [\because K = 2C]$$

Q. 18 Find the general solution of $y^2 dx + (x^2 - xy + y^2)\, dy = 0$.

Sol. Given, differential equation is

$$y^2 dx + (x^2 - xy + y^2)\, dy = 0$$

$$\Rightarrow \quad y^2 dx = -(x^2 - xy + y^2)\, dy$$

$$\Rightarrow \quad y^2 \frac{dx}{dy} = -(x^2 - xy + y^2)$$

$$\Rightarrow \quad \frac{dx}{dy} = -\left(\frac{x^2}{y^2} - \frac{x}{y} + 1\right) \qquad \dots (i)$$

which is a homogeneous differential equation.

Put $\frac{x}{y} = v$ or $x = vy$

$$\Rightarrow \quad \frac{dx}{dy} = v + y\frac{dv}{dy}$$

On substituting these values in Eq. (i), we get

$$v + y\frac{dv}{dy} = -[v^2 - v + 1]$$

$$\Rightarrow \quad y\frac{dv}{dy} = -v^2 + v - 1 - v$$

$$\Rightarrow \quad y\frac{dv}{dy} = -v^2 - 1 \Rightarrow \frac{dv}{v^2+1} = -\frac{dy}{y}$$

On integrating both sides, we get

$$\tan^{-1}(v) = -\log y + C$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{x}{y}\right) + \log y = C \qquad \left[\because v = \frac{x}{y}\right]$$

Q. 19 Solve $(x + y)(dx - dy) = dx + dy$.

Sol. Given differential equation is

$$(x + y)(dx - dy) = dx + dy$$

$$\Rightarrow \quad (x + y)\left(1 - \frac{dy}{dx}\right) = 1 + \frac{dy}{dx} \qquad \dots (i)$$

Put $x + y = z$

$$\Rightarrow \quad 1 + \frac{dy}{dx} = \frac{dz}{dx}$$

On substituting these values in Eq. (i), we get

$$z\left(1 - \frac{dz}{dx} + 1\right) = \frac{dz}{dx}$$

$$\Rightarrow \quad z\left(2 - \frac{dz}{dx}\right) = \frac{dz}{dx}$$

$$\Rightarrow \quad 2z - z\frac{dz}{dx} - \frac{dz}{dx} = 0$$

$$\Rightarrow \quad 2z - (z+1)\frac{dz}{dx} = 0$$

$$\Rightarrow \quad \frac{dz}{dx} = \frac{2z}{z+1}$$

$$\Rightarrow \quad \left(\frac{z+1}{z}\right) dz = 2\,dx$$

On integrating both sides, we get

$$\int \left(1 + \frac{1}{z}\right) dz = 2\int dx$$

$$\Rightarrow \quad z + \log z = 2x - \log C$$

$$\Rightarrow \quad (x+y) + \log(x+y) = 2x - \log C \qquad [\because z = x+y]$$

$$\Rightarrow \quad 2x - x - y = \log C + \log(x+y)$$

$$\Rightarrow \quad x - y = \log|C(x+y)|$$

$$\Rightarrow \quad e^{x-y} = C(x+y)$$

$$\Rightarrow \quad (x+y) = \frac{1}{C}e^{x-y}$$

$$\Rightarrow \quad x + y = Ke^{x-y} \qquad \left[\because K = \frac{1}{C}\right]$$

Q. 20 Solve $2(y+3) - xy\dfrac{dy}{dx} = 0$, given that $y(1) = -2$.

Sol. Given that,

$$2(y+3) - xy\frac{dy}{dx} = 0$$

$$\Rightarrow \quad 2(y+3) = xy\frac{dy}{dx}$$

$$\Rightarrow \quad 2\frac{dx}{x} = \left(\frac{y}{y+3}\right) dy$$

$$\Rightarrow \quad 2\cdot\frac{dx}{x} = \left(\frac{y+3-3}{y+3}\right) dy$$

$$\Rightarrow \quad 2\cdot\frac{dx}{x} = \left(1 - \frac{3}{y+3}\right) dy$$

On integrating both sides, we get

$$2\log x = y - 3\log(y+3) + C \qquad \text{... (i)}$$

When $x = 1$ and $y = -2$, then

$$2\log 1 = -2 - 3\log(-2+3) + C$$

$$\Rightarrow \quad 2\cdot 0 = -2 - 3\cdot 0 + C$$

$$\Rightarrow \quad C = 2$$

On substituting the value of C in Eq. (i), we get

$$2\log x = y - 3\log(y+3) + 2$$

$$\Rightarrow \quad 2\log x + 3\log(y+3) = y + 2$$

$$\Rightarrow \quad \log x^2 + \log(y+3)^3 = (y+2)$$

$$\Rightarrow \quad \log x^2(y+3)^3 = y + 2$$

$$\Rightarrow \quad x^2(y+3)^3 = e^{y+2}$$

Q. 21 Solve the differential equation $dy = \cos x\,(2 - y\,\text{cosec}\,x)\,dx$ given that $y = 2$, when $x = \frac{\pi}{2}$.

Sol. Given differential equation,

$$dy = \cos x\,(2 - y\,\text{cosec}\,x)dx$$

$$\Rightarrow \quad \frac{dy}{dx} = \cos x\,(2 - y\,\text{cosec}\,x)$$

$$\Rightarrow \quad \frac{dy}{dx} = 2\cos x - y\,\text{cosec}\,x \cdot \cos x$$

$$\Rightarrow \quad \frac{dy}{dx} = 2\cos x - y\cot x$$

$$\Rightarrow \quad \frac{dy}{dx} + y\cot x = 2\cos x$$

which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = \cot x,\ Q = 2\cos x$$

$$\text{IF} = e^{\int P dx} = e^{\int \cot x\,dx} = e^{\log \sin x} = \sin x$$

The general solution is

$$y \cdot \sin x = \int 2\cos x \cdot \sin x\,dx + C$$

$$\Rightarrow \quad y \cdot \sin x = \int \sin 2x\,dx + C \qquad [\because \sin 2x = 2\sin x \cos x]$$

$$\Rightarrow \quad y \cdot \sin x = -\frac{\cos 2x}{2} + C \qquad \text{... (i)}$$

When $x = \frac{\pi}{2}$ and $y = 2$, then

$$2 . \sin\frac{\pi}{2} = -\frac{\cos\left(2 \times \frac{\pi}{2}\right)}{2} + C$$

$$\Rightarrow \quad 2 \cdot 1 = +\frac{1}{2} + C$$

$$\Rightarrow \quad 2 - \frac{1}{2} = C \Rightarrow \frac{4-1}{2} = C$$

$$\Rightarrow \therefore \quad C = \frac{3}{2}$$

On substituting the value of C in Eq. (i), we get

$$y\sin x = -\frac{1}{2}\cos 2x + \frac{3}{2}$$

Q. 22 Form the differential equation by eliminating A and B in $Ax^2 + By^2 = 1$.

Sol. Given differential equation is $\quad Ax^2 + By^2 = 1$

On differentiating both sides w.r.t. x, we get

$$2Ax + 2By\frac{dy}{dx} = 0$$

$$\Rightarrow \quad 2By\frac{dy}{dx} = -2Ax$$

$$\Rightarrow \quad By\frac{dy}{dx} = -Ax \Rightarrow \frac{y}{x} \cdot \frac{dy}{dx} = -\frac{A}{B}$$

Again, differentiating w.r.t. x, we get

$$\frac{y}{x}\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot\left(\frac{x\frac{dy}{dx}-y}{x^2}\right)=0$$

$$\Rightarrow \quad \frac{y}{x}\cdot\frac{d^2y}{dx^2}+\frac{x\left(\frac{dy}{dx}\right)^2-y\left(\frac{dy}{dx}\right)}{x^2}=0$$

$$\Rightarrow \quad xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\left(\frac{dy}{dx}\right)=0$$

$$\Rightarrow \quad xy\,y''+x\,(y')^2-y\,y'=0$$

Q. 23 Solve the differential equation $(1+y^2)\tan^{-1}x\,dx+2y\,(1+x^2)\,dy=0$.

Sol. Given differential equation is

$$(1+y^2)\tan^{-1}x\,dx+2y\,(1+x^2)\,dy=0$$

$$\Rightarrow \quad (1+y^2)\tan^{-1}x\,dx=-2\,y\,(1+x^2)\,dy$$

$$\Rightarrow \quad \frac{\tan^{-1}x\,dx}{1+x^2}=-\frac{2\,y}{1+y^2}dy$$

On integrating both sides, we get

$$\int\frac{\tan^{-1}x}{1+x^2}dx=-\int\frac{2y}{1+y^2}dy$$

Put $\tan^{-1}x=t$ in LHS, we get

$$\frac{1}{1+x^2}dx=dt$$

and put $1+y^2=u$ in RHS, we get

$$2\,y\,dy=du$$

$$\Rightarrow \quad \int t\,dt=-\int\frac{1}{u}du \Rightarrow \frac{t^2}{2}=-\log u+C$$

$$\Rightarrow \quad \frac{1}{2}(\tan^{-1}x)^2=-\log(1+y^2)+C$$

$$\Rightarrow \quad \frac{1}{2}(\tan^{-1}x)^2+\log(1+y^2)=C$$

Q. 24 Find the differential equation of system of concentric circles with centre (1, 2).

Sol. The family of concentric circles with centre (1, 2) and radius a is given by

$$(x-1)^2+(y-2)^2=a^2$$

$$\Rightarrow \quad x^2+1-2x+y^2+4-4y=a^2$$

$$\Rightarrow \quad x^2+y^2-2x-4y+5=a^2 \quad \text{...(i)}$$

On differentiating Eq. (i) w.r.t. x, we get

$$2x+2y\frac{dy}{dx}-2-4\frac{dy}{dx}=0$$

$$\Rightarrow \quad (2y-4)\frac{dy}{dx}+2x-2=0$$

$$\Rightarrow \quad (y-2)\frac{dy}{dx}+(x-1)=0$$

Long Answer Type Questions

Q. 25 Solve $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$.

Sol. Given differential equation is

$$y + \frac{d}{dx}(xy) = x(\sin x + \log x)$$

$$\Rightarrow \quad y + x\frac{dy}{dx} + y = x(\sin x + \log x)$$

$$\Rightarrow \quad x\frac{dy}{dx} + 2y = x(\sin x + \log x)$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{2}{x}y = \sin x + \log x$$

which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = \frac{2}{x}, \ Q = \sin x + \log x$$

$$\text{IF} = e^{\int \frac{2}{x}dx} = e^{2\log x} = x^2$$

The general solution is

$$y \cdot x^2 = \int (\sin x + \log x)\, x^2 dx + C$$

$$\Rightarrow \quad y \cdot x^2 = \int (x^2 \sin x + x^2 \log x)\, dx + C$$

$$\Rightarrow \quad y \cdot x^2 = \int x^2 \sin x \, dx + \int x^2 \log x \, dx + C$$

$$\Rightarrow \quad y \cdot x^2 = I_1 + I_2 + C \quad \text{...(i)}$$

Now,

$$I_1 = \int x^2 \sin x \, dx$$

$$= x^2(-\cos x) + \int 2x \cos x \, dx$$

$$= -x^2 \cos x + [2x(\sin x) - \int 2\sin x \, dx]$$

$$I_1 = -x^2 \cos x + 2x \sin x + 2\cos x \quad \text{... (ii)}$$

and

$$I_2 = \int x^2 \log x \, dx$$

$$= \log x \cdot \frac{x^3}{3} - \int \frac{1}{x} \cdot \frac{x^3}{3} dx$$

$$= \log x \cdot \frac{x^3}{3} - \frac{1}{3}\int x^2 dx$$

$$= \log x \cdot \frac{x^3}{3} - \frac{1}{3} \cdot \frac{x^3}{3} \quad \text{... (iii)}$$

On substituting the value of I_1 and I_2 in Eq. (i), we get

$$y \cdot x^2 = -x^2 \cos x + 2x \sin x + 2\cos x + \frac{x^3}{3}\log x - \frac{1}{9}x^3 + C$$

$$\therefore \quad y = -\cos x + \frac{2\sin x}{x} + \frac{2\cos x}{x^2} + \frac{x}{3}\log x - \frac{x}{9} + Cx^{-2}$$

Q. 26 Find the general solution of $(1+\tan y)(dx - dy) + 2x\,dy = 0$.

Sol. Given differential equation is $(1+\tan y)(dx - dy) + 2x\,dy = 0$
on dividing throughout by dy, we get

$$(1+\tan y)\left(\frac{dx}{dy} - 1\right) + 2x = 0$$

$$\Rightarrow \quad (1+\tan y)\frac{dx}{dy} - (1+\tan y) + 2x = 0$$

$$\Rightarrow \quad (1+\tan y)\frac{dx}{dy} + 2x = (1+\tan y)$$

$$\Rightarrow \quad \frac{dx}{dy} + \frac{2x}{1+\tan y} = 1$$

which is a linear differential equation.

On comparing it with $\frac{dx}{dy} + Px = Q$, we get

$$P = \frac{2}{1+\tan y}, Q = 1$$

$$\text{IF} = e^{\int \frac{2}{1+\tan y} dy} = e^{\int \frac{2\cos y}{\cos y + \sin y} dy}$$

$$= e^{\int \frac{\cos y + \sin y + \cos y - \sin y}{\cos y + \sin y} dy}$$

$$= e^{\int \left(1 + \frac{\cos y - \sin y}{\cos y + \sin y}\right) dy} = e^{y + \log(\cos y + \sin y)}$$

$$= e^{y} \cdot (\cos y + \sin y) \qquad [\because e^{\log x} = x]$$

The general solution is

$$x \cdot e^{y}(\cos y + \sin y) = \int 1 \cdot e^{y}(\cos y + \sin y)\,dy + C$$

$$\Rightarrow \quad x \cdot e^{y}(\cos y + \sin y) = \int e^{y}(\sin y + \cos y)\,dy + C$$

$$\Rightarrow \quad x \cdot e^{y}(\cos y + \sin y) = e^{y}\sin y + C \qquad \left[\because \int e^{x}\{f(x) + f'(x)\}\,dx = e^{x} f(x)\right]$$

$$\Rightarrow \quad x(\sin y + \cos y) = \sin y + Ce^{-y}$$

Q. 27 Solve $\frac{dy}{dx} = \cos(x+y) + \sin(x+y)$.

Sol. Given, $$\frac{dy}{dx} = \cos(x+y) + \sin(x+y) \qquad \ldots(\text{i})$$

Put $x + y = z$

$$\Rightarrow \quad 1 + \frac{dy}{dx} = \frac{dz}{dx}$$

On substituting these values in Eq. (i), we get

$$\left(\frac{dz}{dx} - 1\right) = \cos z + \sin z$$

$$\Rightarrow \quad \frac{dz}{dx} = (\cos z + \sin z + 1)$$

$$\Rightarrow \quad \frac{dz}{\cos z + \sin z + 1} = dx$$

On integrating both sides, we get

$$\int \frac{dz}{\cos z + \sin z + 1} = \int 1 d\,x$$

$$\Rightarrow \quad \int \frac{dz}{\frac{1 - \tan^2 z/2}{1 + \tan^2 z/2} + \frac{2 \tan z/2}{1 + \tan^2 z/2} + 1} = \int dx$$

$$\Rightarrow \quad \int \frac{dz}{\frac{1 - \tan^2 z/2 + 2 \tan z/2 + 1 + \tan^2 z/2}{(1 + \tan^2 z/2)}} = \int dx$$

$$\Rightarrow \quad \int \frac{(1 + \tan^2 z/2)\, dz}{2 + 2 \tan^2 z/2} = \int dx$$

$$\Rightarrow \quad \int \frac{\sec^2 z/2\, dz}{2\,(1 + \tan z/2)} = \int dx$$

Put $1 + \tan z/2 = t \Rightarrow$ $\left(\frac{1}{2} \sec^2 z/2\right) d\,z = d\,t$

$$\Rightarrow \quad \int \frac{dt}{t} = \int dx$$

$$\Rightarrow \quad \log|t| = x + C$$

$$\Rightarrow \quad \log|1 + \tan z/2| = x + C$$

$$\Rightarrow \quad \log\left|1 + \tan\frac{(x + y)}{2}\right| = x + C$$

Q. 28 Find the general solution of $\frac{dy}{dx} - 3y = \sin 2x$.

Sol. Given, $\frac{dy}{dx} - 3y = \sin 2x$

which is a linear differential equation.

On comparing it with $\frac{dy}{dx} + Py = Q$, we get

$$P = -3, Q = \sin 2x$$

$$\text{IF} = e^{-3\int dx} = e^{-3x}$$

The general solution is

$$y\,.\,e^{-3x} = \int \underset{\text{I}}{\sin 2x}\, \underset{\text{II}}{e^{-3x}}\, dx$$

Let $y \cdot e^{-3x} = I$ ... (i)

$$\therefore \quad I = \int \underset{\text{II}}{e^{-3x}}\, \underset{\text{I}}{\sin 2x}$$

$$\Rightarrow \quad I = \sin 2x \left(.\frac{e^{-3x}}{-3}\right) - \int 2 \cos 2x \left(\frac{e^{-3x}}{-3}\right) dx + C_1$$

$$\Rightarrow \quad I = -\frac{1}{3} e^{-3x} \sin 2x + \frac{2}{3} \int \underset{\text{II}}{e^{-3x}}\, \underset{\text{I}}{\cos 2x}\, dx + C_1$$

$$\Rightarrow \quad I = -\frac{1}{3} e^{-3x} \sin 2x + \frac{2}{3} \left(\cos 2x\, \frac{e^{-3x}}{-3} - \int (-2 \sin 2x) \frac{e^{-3x}}{-3} dx\right) + C_1 + C_2$$

$$\Rightarrow \quad I = -\frac{1}{3} e^{-3x} \sin 2x - \frac{2}{9} \cos 2x\, e^{-3x} - \frac{4}{9} I + C' \qquad [\text{where, } C' = C_1 + C_2]$$

$$\Rightarrow \quad I + \frac{4I}{9} 2 = + e^{-3x} \left(-\frac{1}{3} \sin 2x - \frac{2}{9} \cos 2x\right) + C'$$

$$\Rightarrow \quad \frac{13}{9} I = e^{-3x}\left(-\frac{1}{3}\sin 2x - \frac{2}{9}\cos 2x\right) + C'$$

$$\Rightarrow \quad I = \frac{9}{13} e^{-3x}\left(-\frac{1}{3}\sin 2x - \frac{2}{9}\cos 2x\right) + C \qquad \left[\text{where } C = \frac{9C'}{13}\right]$$

$$\Rightarrow \quad I = \frac{3}{13} e^{-3x}\left(-\sin 2x - \frac{2}{3}\cos 2x\right) + C$$

$$\Rightarrow \quad = \frac{3}{13} e^{-3x}\frac{(-3\sin 2x - 2\cos 2x)}{3} + C$$

$$\Rightarrow \quad = \frac{e^{-3x}}{13}(-3\sin 2x - 2\cos 2x) + C$$

$$\Rightarrow \quad I = -\frac{e^{-3x}}{13}(2\cos 2x + 3\sin 2x) + C$$

On substituting the value of I in Eq. (i), we get

$$y \cdot e^{-3x} = -\frac{e^{-3x}}{13}(2\cos 2x + 3\sin 2x) + C$$

$$\Rightarrow \quad y = -\frac{1}{13}(2\cos 2x + 3\sin 2x) + C e^{3x}$$

Q. 29 Find the equation of a curve passing through (2, 1), if the slope of the tangent to the curve at any point (x, y) is $\dfrac{x^2 + y^2}{2xy}$.

Sol. It is given that, the slope of tangent to the curve at point (x, y) is $\dfrac{x^2 + y^2}{2xy}$.

$$\therefore \quad \left(\frac{dy}{dx}\right)_{(x, y)} = \frac{x^2 + y^2}{2xy}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{1}{2}\left(\frac{x}{y} + \frac{y}{x}\right) \qquad \ldots \text{(i)}$$

which is homogeneous differential equation.

Put $\quad y = vx$

$$\Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

On substituting these values in Eq. (i), we get

$$v + x\frac{dv}{dx} = \frac{1}{2}\left(\frac{1}{v} + v\right)$$

$$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{1}{2}\left(\frac{1 + v^2}{v}\right)$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1 + v^2}{2v} - v$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1 + v^2 - 2v^2}{2v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1 - v^2}{2v}$$

$$\Rightarrow \quad \frac{2v}{1 - v^2}dv = \frac{dx}{x}$$

On integrating both sides, we get

$$\int \frac{2v}{1-v^2}\,dv = \int \frac{dx}{x}$$

Put $1 - v^2 = t$ in LHS, we get

$$-2\,v dv = dt$$

$$\Rightarrow \quad -\int \frac{dt}{t} = \int \frac{dx}{x}$$

$$\Rightarrow \quad -\log t = \log x + \log C$$

$$\Rightarrow \quad -\log(1 - v^2) = \log x + \log C$$

$$\Rightarrow \quad -\log\left(1 - \frac{y^2}{x^2}\right) = \log x + \log C$$

$$\Rightarrow \quad -\log\left(\frac{x^2 - y^2}{x^2}\right) = \log x + \log C$$

$$\Rightarrow \quad \log\left(\frac{x^2}{x^2 - y^2}\right) = \log x + \log C$$

$$\Rightarrow \quad \frac{x^2}{x^2 - y^2} = C\,x \qquad \ldots(\text{ii})$$

Since, the curve passes through the point (2, 1).

$$\therefore \quad \frac{(2)^2}{(2)^2 - (1)^2} = C\,(2) \;\Rightarrow C = \frac{2}{3}$$

So, the required solution is $2\,(x^2 - y^2) = 3x$.

Q. 30 Find the equation of the curve through the point (1, 0), if the slope of the tangent to the curve at any point (x, y) is $\dfrac{y-1}{x^2 + x}$.

Sol. It is given that, slope of tangent to the curve at any point (x, y) is $\dfrac{y-1}{x^2+x}$.

$$\therefore \quad \left(\frac{dy}{dx}\right)_{(x,\,y)} = \frac{y-1}{x^2+x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y-1}{x^2+x}$$

$$\Rightarrow \quad \frac{dy}{y-1} = \frac{dx}{x^2+x}$$

On integrating both sides, we get

$$\int \frac{dy}{y-1} = \int \frac{dx}{x^2+x}$$

$$\Rightarrow \quad \int \frac{dy}{y-1} = \int \frac{dx}{x\,(x+1)}$$

$$\Rightarrow \quad \int \frac{dy}{y-1} = \int \left(\frac{1}{x} - \frac{1}{x+1}\right) dx$$

$$\Rightarrow \quad \log(y-1) = \log x - \log(x+1) + \log C$$

$$\Rightarrow \quad \log(y-1) = \log\left(\frac{x\,C}{x+1}\right)$$

Since, the given curve passes through point (1, 0).

$$\therefore \quad 0 - 1 = \frac{1 \cdot C}{1 + 1} \Rightarrow C = -2$$

The particular solution is $\quad y - 1 = \dfrac{-2x}{x + 1}$

$$\Rightarrow \quad (y - 1)(x + 1) = -2x$$

$$\Rightarrow \quad (y - 1)(x + 1) + 2x = 0$$

Q. 31 Find the equation of a curve passing through origin, if the slope of the tangent to the curve at any point (x, y) is equal to the square of the difference of the abcissa and ordinate of the point.

Sol. Slope of tangent to the curve $= \dfrac{dy}{dx}$

and difference of abscissa and ordinate $= x - y$

According to the question, $\quad \dfrac{dy}{dx} = (x - y)^2 \quad \ldots(i)$

Put $\quad x - y = z$

$$\Rightarrow \quad 1 - \frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow \quad \frac{dy}{dx} = 1 - \frac{dz}{dx}$$

On substituting these values in Eq. (i), we get

$$1 - \frac{dz}{dx} = z^2$$

$$\Rightarrow \quad 1 - z^2 = \frac{dz}{dx}$$

$$\Rightarrow \quad dx = \frac{dz}{1 - z^2}$$

On integrating both sides, we get

$$\int dx = \int \frac{dz}{1 - z^2}$$

$$\Rightarrow \quad x = \frac{1}{2}\log\left|\frac{1 + z}{1 - z}\right| + C$$

$$\Rightarrow \quad \text{t}x = \frac{1}{2}\log\left|\frac{1 + x - y}{1 - x + y}\right| + C \quad \ldots(ii)$$

Since, the curve passes through the origin.

$$\therefore \quad 0 = \frac{1}{2}\log\left|\frac{1 + 0 - 0}{1 - 0 + 0}\right| + C$$

$$\Rightarrow \quad C = 0$$

On substituting the value of C in Eq. (ii), we get

$$x = \frac{1}{2}\log\left|\frac{1 + x - y}{1 - x + y}\right|$$

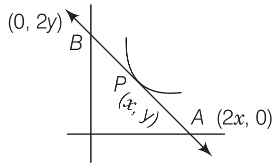
$$\Rightarrow \quad 2x = \log\left|\frac{1 + x - y}{1 - x + y}\right|$$

$$\Rightarrow \quad e^{2x} = \left|\frac{1 + x - y}{1 - x + y}\right|$$

$$\Rightarrow \quad (1 - x + y)e^{2x} = 1 + x - y$$

Q. 32 Find the equation of a curve passing through the point (1, 1), if the tangent drawn at any point $P(x, y)$ on the curve meets the coordinate axes at A and B such that P is the mid-point of AB.

Sol. The below figure obtained by the given information



Let the coordinate of the point P is (x, y). It is given that, P is mid-point of AB.
So, the coordinates of points A and B are $(2x, 0)$ and $(0, 2y)$, respectively.

$$\therefore \quad \text{Slope of } AB = \frac{0-2y}{2x-0} = -\frac{y}{x}$$

Since, the segment AB is a tangent to the curve at P.

$$\therefore \quad \frac{dy}{dx} = -\frac{y}{x}$$

$$\Rightarrow \quad \frac{dy}{y} = -\frac{dx}{x}$$

On integrating both sides, we get

$$\log y = -\log x + \log C$$

$$\log y = \log \frac{C}{x} \qquad \ldots(i)$$

Since, the given curve passes through (1, 1).

$$\therefore \quad \log 1 = \log \frac{C}{1}$$

$$\Rightarrow \quad 0 = \log C$$

$$\Rightarrow \quad c = 1$$

$$\therefore \quad \log y = \log \frac{1}{x}$$

$$\Rightarrow \quad y = \frac{1}{x}$$

$$\Rightarrow \quad xy = 1$$

Q. 33 Solve $x\dfrac{dy}{dx} = y(\log y - \log x + 1)$

Sol. Given,

$$x\frac{dy}{dx} = y(\log y - \log x + 1)$$

$$\Rightarrow \quad x\frac{dy}{dx} = y\log\left(\frac{y}{x} + 1\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right) \qquad \ldots(i)$$

which is a homogeneous equation.

Put $\quad \dfrac{y}{x} = v$ or $y = vx$

$$\therefore \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

On substituting these values in Eq.(i), we get

$$v + x\frac{dv}{dx} = v(\log v + 1)$$

$$\Rightarrow \quad x\frac{dv}{dx} = v(\log v + 1 - 1)$$

$$\Rightarrow \quad x\frac{dv}{dx} = v(\log v)$$

$$\Rightarrow \quad \frac{dv}{v\log v} = \frac{dx}{x}$$

On integrating both sides, we get

$$\int \frac{dv}{v\log v} = \int \frac{dx}{x}$$

On putting $\log v = u$ in LHS integral, we get

$$\frac{1}{v} \cdot dv = du$$

$$\int \frac{du}{u} = \int \frac{dx}{x}$$

$$\Rightarrow \quad \log u = \log x + \log C$$

$$\Rightarrow \quad \log u = \log C\, x$$

$$\Rightarrow \quad u = Cx$$

$$\Rightarrow \quad \log v = Cx$$

$$\Rightarrow \quad \log\left(\frac{y}{x}\right) = Cx$$

Objective Type Questions

Q. 34 The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{dy}{dx}\right)$ is

(a) 1 (b) 2 (c) 3 (d) not defined

Sol. *(d)* The degree of above differential equation is not defined because when we expand $\sin\left(\frac{dy}{dx}\right)$ we get an infinite series in the increasing powers of $\frac{dy}{dx}$. Therefore its degree is not defined.

Q. 35 The degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$ is

(a) 4 (b) $\frac{3}{2}$ (c) not defined (d) 2

Sol. *(d)* Given that $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = \frac{d^2y}{dx^2}$

On squaring both sides, we get

$$\left[1 + \left(\frac{dy}{dx}\right)^2\right]^3 = \left(\frac{d^2y}{dx^2}\right)^2$$

So, the degree of differential equation is 2.

Q. 36 The order and degree of the differential equation $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/4}+x^{1/5}=0$ respectively, are

(a) 2 and 4 (b) 2 and 2
(c) 2 and 3 (d) 3 and 3

Sol. *(a)* Given that,
$$\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/4}=-x^{1/5}$$
$$\Rightarrow \quad \frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^{1/4}=-x^{1/5}$$
$$\Rightarrow \quad \left(\frac{dy}{dx}\right)^{1/4}=-\left(x^{1/5}+\frac{d^2y}{dx^2}\right)$$

On squaring both sides, we get
$$\left(\frac{dy}{dx}\right)^{1/2}=\left(x^{1/5}+\frac{d^2y}{dx^2}\right)^2$$

Again, on squaring both sides, we have
$$\frac{dy}{dx}=\left(x^{1/5}+\frac{d^2y}{dx^2}\right)^4$$

order = 2, degree = 4

Q. 37 If $y=e^{-x}(A\cos x+B\sin x)$, then y is a solution of

(a) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}=0$ (b) $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=0$

(c) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$ (d) $\frac{d^2y}{dx^2}+2y=0$

Sol. *(c)* Given that, $y=e^{-x}(A\cos x+B\sin x)$

On differentiating both sides w.r.t., x we get
$$\frac{dy}{dx}=-e^{-x}(A\cos x+B\sin x)+e^{-x}(-A\sin x+B\cos x)$$
$$\frac{dy}{dx}=-y+e^{-x}(-A\sin x+B\cos x)$$

Again, differentiating both sides w.r.t. x, we get
$$\frac{d^2y}{dx^2}=\frac{-dy}{dx}+e^{-x}(-\cos x-B\sin x)-e^{-x}(-A\sin x+B\cos x)$$
$$\Rightarrow \quad \frac{d^2y}{dx^2}=-\frac{dy}{dx}-y-\left[\frac{dy}{dx}+y\right]$$
$$\Rightarrow \quad \frac{d^2y}{dx^2}=-\frac{dy}{dx}-y-\frac{dy}{dx}-y$$
$$\Rightarrow \quad \frac{d^2y}{dx^2}=-2\frac{dy}{dx}-2y$$
$$\Rightarrow \quad \frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$$

Q. 38 The differential equation for $y = A\cos\alpha x + B\sin\alpha x$, where A and B are arbitrary constants is

(a) $\frac{d^2y}{dx^2} - \alpha^2 y = 0$ (b) $\frac{d^2y}{dx^2} + \alpha^2 y = 0$

(c) $\frac{d^2y}{dx^2} + \alpha y = 0$ (d) $\frac{d^2y}{dx^2} - \alpha y = 0$

Sol. *(b)* Given, $y = A\cos\alpha + B\sin\alpha$

$\Rightarrow \frac{dy}{dx} = -\alpha A\sin\alpha x + \alpha B\cos\alpha x$

Again, differentiating both sides w.r.t. x, we get

$\frac{d^2y}{dx^2} = -A\alpha^2\cos\alpha x - \alpha^2 B\sin\alpha x$

$\Rightarrow \frac{d^2y}{dx^2} = -\alpha^2(A\cos\alpha x + B\sin\alpha x)$

$\Rightarrow \frac{d^2y}{dx^2} = -\alpha^2 y$

$\Rightarrow \frac{d^2y}{dx^2} + \alpha^2 y = 0$

Q. 39 The solution of differential equation $xdy - ydx = 0$ represents

(a) a rectangular hyperbola
(b) parabola whose vertex is at origin
(c) straight line passing through origin
(d) a circle whose centre is at origin

Sol. *(c)* Given that, $xdy - ydx = 0$

$\Rightarrow xdy = ydx$

$\Rightarrow \frac{dy}{y} = \frac{dx}{x}$

On integrating both sides, we get

$\log y = \log x + \log C$

$\Rightarrow \log y = \log Cx$

$\Rightarrow y = Cx$

which is a straight line passing through origin.

Q. 40 The integrating factor of differential equation $\cos x\frac{dy}{dx} + y\sin x = 1$ is

(a) $\cos x$ (b) $\tan x$ (c) $\sec x$ (d) $\sin x$

Sol. *(c)* Given that, $\cos x\frac{dy}{dx} + y\sin x = 1$

$\Rightarrow \frac{dy}{dx} + y\tan x = \sec x$

Here, $P = \tan x$ and $Q = \sec x$

$\text{IF} = e^{\int P dx} = e^{\int \tan x dx} = e^{\log \sec x}$

$\therefore = \sec x$

Q. 41 The solution of differential equation $\tan y \sec^2 x dx + \tan x \sec^2 y dy = 0$ is

(a) $\tan x + \tan y = k$ (b) $\tan x - \tan y = k$
(c) $\frac{\tan x}{\tan y} = k$ (d) $\tan x \cdot \tan y = k$

Sol. *(d)* Given that,
$$\tan y \sec^2 x dx + \tan x \sec^2 y dy = 0$$
$$\Rightarrow \quad \tan \sec^2 x dx = -\tan x \sec^2 y dy$$
$$\Rightarrow \quad \frac{\sec^2 x}{\tan x} dx = \frac{-\sec^2 y}{\tan y} dy \quad \ldots(i)$$

On integrating both sides, we have
$$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 y}{\tan y} dy$$

Put $\tan x = t$ in LHS integral, we get
$$\sec^2 x\, dx = dt \Rightarrow \sec^2 x\, dx = dt$$

and $\tan y = u$ in RHS integral, we get
$$\sec^2 y dy = du$$

On substituting these values in Eq. (i), we get
$$\int \frac{dt}{t} = -\int \frac{du}{u}$$
$$\log t = -\log u + \log k$$
$$\Rightarrow \quad \log(t \cdot u) = \log k$$
$$\Rightarrow \quad \log(\tan x \tan y) = \log k$$
$$\Rightarrow \quad \tan x \tan y = k$$

Q. 42 The family $y = Ax + A^3$ of curves is represented by differential equation of degree

(a) 1 (b) 2 (c) 3 (d) 4

Sol. *(a)* Given that,
$$y = Ax + A^3$$
$$\Rightarrow \quad \frac{dy}{dx} = A$$

[we can differential above equation only once because it has only one arbitrary constant]

$\therefore$ Degree = 1

Q. 43 The integrating factor of $\frac{xdy}{dx} - y = x^4 - 3x$ is

(a) x (b) $\log x$ (c) $\frac{1}{x}$ (d) $-x$

Sol. *(c)* Given that,
$$x\frac{dy}{dx} - y = x^4 - 3x$$
$$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = x^3 - 3$$

Here, $P = -\frac{1}{x}, Q = x^3 - 3$

$$\therefore \quad \text{IF} = e^{\int P dx} = e^{-\int \frac{1}{x} dx} = e^{-\log x}$$
$$= \frac{1}{x}$$

Q. 44 The solution of $\frac{dy}{dx} - y = 1,\ y(0) = 1$ is given by

(a) $xy = -e^x$
(b) $xy = -e^{-x}$
(c) $xy = -1$
(d) $y = 2e^x - 1$

Sol. *(b)* Given that,

$$\frac{dy}{dx} - y = 1$$

$$\Rightarrow \quad \frac{dy}{dx} = 1 + y$$

$$\Rightarrow \quad \frac{dy}{1+y} = dx$$

On integrating both sides, we get

$$\log(1+y) = x + C \quad \text{...(i)}$$

When $x = 0$ and $y = 1$, then

$$\log 2 = 0 + c$$

$$\Rightarrow \quad C = \log 2$$

The required solution is

$$\log(1+y) = x + \log 2$$

$$\Rightarrow \quad \log\left(\frac{1+y}{2}\right) = x$$

$$\Rightarrow \quad \frac{1+y}{2} = e^x$$

$$\Rightarrow \quad 1 + y = 2e^x$$

$$\Rightarrow \quad y = 2e^x - 1$$

Q. 45 The number of solutions of $\frac{dy}{dx} = \frac{y+1}{x-1}$, when $y(1) = 2$ is

(a) none
(b) one
(c) two
(d) infinite

Sol. *(b)* Given that, $\frac{dy}{dx} = \frac{y+1}{x-1}$

$$\Rightarrow \quad \frac{dy}{y+1} = \frac{dx}{x-1}$$

On integrating both sides, we get

$$\log(y+1) = \log(x-1) - \log C$$

$$C(y+1) = (x-1)$$

$$\Rightarrow \quad C = \frac{x-1}{y+1}$$

When $x = 1$ and $y = 2$, then $C = 0$

So, the required solution is $x - 1 = 0$.

Hence, only one solution exist.

Q. 46 Which of the following is a second order differential equation?

(a) $(y')^2 + x = y^2$
(b) $y'y'' + y = \sin x$
(c) $y''' + (y'')^2 + y = 0$
(d) $y' = y^2$

Sol. *(b)* The second order differential equation is $y'y'' + y = \sin x$.

Q. 47 The integrating factor of differential equation $(1-x^2)\dfrac{dy}{dx} - xy = 1$ is

(a) $-x$ (b) $\dfrac{x}{1+x^2}$ (c) $\sqrt{1-x^2}$ (d) $\dfrac{1}{2}\log(1-x^2)$

Sol. *(c)* Given that, $$(1-x^2)\frac{dy}{dx} - xy = 1$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{x}{1-x^2}y = \frac{1}{1-x^2}$$

which is a linear differential equation.

$$\therefore \quad \text{IF} = e^{-\int \frac{x}{1-x^2}dx}$$

Put $$1-x^2 = t \Rightarrow -2xdx = dt \Rightarrow xdx = -\frac{dt}{2}$$

Now, $$\text{IF} = e^{\frac{1}{2}\int\frac{dt}{t}} = e^{\frac{1}{2}\log t} = e^{\frac{1}{2}\log(1-x^2)} = \sqrt{1-x^2}$$

Q. 48 $\tan^{-1}x + \tan^{-1}y = C$ is general solution of the differential equation

(a) $\dfrac{dy}{dx} = \dfrac{1+y^2}{1+x^2}$ (b) $\dfrac{dy}{dx} = \dfrac{1+x^2}{1+y^2}$

(c) $(1+x^2)dy + (1+y^2)dx = 0$ (d) $(1+x^2)dx + (1+y^2)dy = 0$

Sol. *(c)* Given that, $\tan^{-1}x + \tan^{-1}y = C$

On differentiating w.r.t. x, we get

$$\frac{1}{1+x^2} + \frac{1}{1+y^2}\cdot\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{1}{1+y^2}\cdot\frac{dy}{dx} = -\frac{1}{1+x^2}$$

$$\Rightarrow \quad (1+x^2)\,dy + (1+y^2)\,dx = 0$$

Q. 49 The differential equation $y\dfrac{dy}{dx} + x = C$ represents

(a) family of hyperbolas (b) family of parabolas
(c) family of ellipses (d) family of circles

Sol. *(d)* Given that, $$y\frac{dy}{dx} + x = C$$

$$\Rightarrow \quad y\frac{dy}{dx} = C - x$$

$$\Rightarrow \quad y\,dy = (C-x)\,dx$$

On integrating both sides, we get

$$\frac{y^2}{2} = Cx - \frac{x^2}{2} + K$$

$$\Rightarrow \quad \frac{x^2}{2} + \frac{y^2}{2} = Cx + K$$

$$\Rightarrow \quad \frac{x^2}{2} + \frac{y^2}{2} - Cx = K$$

which represent family of circles.

Q. 50 The general solution of $e^x \cos y dx - e^x \sin y dy = 0$ is

(a) $e^x \cos y = k$ (b) $e^x \sin y = k$

(c) $e^x = k \cos y$ (d) $e^x = k \sin y$

Sol. *(a)* Given that, $e^x \cos y dx - e^x \sin y dy = 0$

$\Rightarrow \quad e^x \cos y dx = e^x \sin y dy$

$\Rightarrow \quad \dfrac{dx}{dy} = \tan y$

$\Rightarrow \quad dx = \tan y dy$

On integrating both sides, we get

$$x = \log \sec y + C$$

$\Rightarrow \quad x - C = \log \sec y$

$\Rightarrow \quad \sec y = e^{x-C}$

$\Rightarrow \quad \sec y = e^x e^{-C}$

$\Rightarrow \quad \dfrac{1}{\cos y} = \dfrac{e^x}{e^C}$

$\Rightarrow \quad e^x \cos y = e^C$

$\Rightarrow \quad e^x \cos y = K$ [where, $K = e^C$]

Q. 51 The degree of differential equation $\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^3 + 6y^5 = 0$ is

(a) 1 (b) 2 (c) 3 (d) 5

Sol. *(a)* $\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^3 + 6y^5 = 0$

We know that, the degree of a differential equation is exponent heighest of order derivative.

$\therefore \quad$ Degree = 1

Q. 52 The solution of $\dfrac{dy}{dx} + y = e^{-x}$, $y(0) = 0$ is

(a) $y = e^x(x-1)$ (b) $y = xe^{-x}$

(c) $y = xe^{-x} + 1$ (d) $y = (x+1)e^{-x}$

Sol. *(b)* Given that, $\dfrac{dy}{dx} + y = e^{-x}$

Here, $P = 1, Q = e^{-x}$

$$\text{IF} = e^{\int P dx} = e^{\int dx} = e^x$$

The general solution is

$$y \cdot e^x = \int e^{-x} e^x dx + C$$

$\Rightarrow \quad y \cdot e^x = \int dx + C$

$\Rightarrow \quad y \cdot e^x = x + C$ ...(i)

When $x = 0$ and $y = 0$, then

$$0 = 0 + C \Rightarrow C = 0$$

Eq. (i) becomes $\quad y \cdot e^x = x$

$\Rightarrow \quad y = xe^{-x}$

Q. 53 The integrating factor of differential equation $\frac{dy}{dx} + y\tan x - \sec x = 0$ is

(a) $\cos x$ (b) $\sec x$ (c) $e^{\cos x}$ (d) $e^{\sec x}$

Sol. *(b)* Given that, $\frac{dy}{dx} + y\tan x - \sec x = 0$

Here, $P = \tan x, Q = \sec x$

$$\text{IF} = e^{\int P dx} = e^{\int \tan x dx}$$
$$= e^{(\log \sec x)}$$
$$= \sec x$$

Q. 54 The solution of differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is

(a) $y = \tan^{-1} x$ (b) $y - x = k(1+xy)$
(c) $x = \tan^{-1} y$ (d) $\tan(xy) = k$

Sol. *(b)* Given that, $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$

$\Rightarrow \frac{dy}{1+y^2} = \frac{dx}{1+x^2}$

On integrating both sides, we get

$$\tan^{-1} y = \tan^{-1} x + C$$

$\Rightarrow \tan^{-1} y - \tan^{-1} x = C$

$\Rightarrow \tan^{-1}\left(\frac{y-x}{1+xy}\right) = C$

$\Rightarrow \frac{y-x}{1+xy} = \tan C$

$\Rightarrow y - x = \tan c(1 + xy)$

$\Rightarrow y - x = K(1 + xy)$

where, $k = \tan C$

Q. 55 The integrating factor of differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is

(a) $\frac{x}{e^x}$ (b) $\frac{e^x}{x}$ (c) xe^x (d) e^x

Sol. *(b)* Given that, $\frac{dy}{dx} + y = \frac{1+y}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1+y}{x} - y$

$\Rightarrow \frac{dy}{dx} = \frac{1+y-xy}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x} + \frac{y(1-x)}{x}$

$\Rightarrow \frac{dy}{dx} - \left(\frac{1-x}{x}\right)y = \frac{1}{x}$

Here, $P = \frac{-(1-x)}{x}, Q = \frac{1}{x}$

$$\text{IF} = e^{\int P dx} = e^{-\int \frac{1-x}{x} dx} = e^{\int \frac{x-1}{x} dx}$$
$$= e^{\int \left(1 - \frac{1}{x}\right) dx}$$
$$= e^{\int x - \log x}$$
$$= e^x \cdot e^{\log\left(\frac{1}{x}\right)}$$
$$= e^x \cdot \frac{1}{x}$$

Q. 56 $y = ae^{mx} + be^{-mx}$ satisfies which of the following differential equation?

(a) $\frac{dy}{dx} + my = 0$

(b) $\frac{dy}{dx} - my = 0$

(c) $\frac{d^2y}{dx^2} - m^2y = 0$

(d) $\frac{d^2y}{dx^2} + m^2y = 0$

Sol. *(c)* Given that, $y = ae^{mx} + be^{-mx}$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = mae^{mx} - bme^{-mx}$$

Again, differentiating both sides w.r.t. x, we get

$$\frac{d^2y}{dx^2} = m^2ae^{mx} + bm^2e^{-mx}$$

$$\Rightarrow \frac{d^2y}{dx^2} = m^2(ae^{mn} + be^{-mn})$$

$$\Rightarrow \frac{d^2y}{dx^2} = m^2y$$

$$\Rightarrow \frac{d^2y}{dx^2} - m^2y = 0$$

Q. 57 The solution of differential equation $\cos x \sin y dx + \sin x \cos y dy = 0$ is

(a) $\frac{\sin x}{\sin y} = C$

(b) $\sin x \sin y = C$

(c) $\sin x + \sin y = C$

(d) $\cos x \cos y = C$

Sol. *(b)* Given differential equation is

$$\cos x \sin y dx + \sin x \cos y dy = 0$$

$$\Rightarrow \cos x \sin y dx = -\sin x \cos y dy$$

$$\Rightarrow \frac{\cos x}{\sin x} dx = -\frac{\cos y}{\sin y} dy$$

$$\Rightarrow \cot x\, dx = -\cot y dy$$

On integrating both sides, we get

$$\log \sin x = -\log \sin y + \log C$$

$$\Rightarrow \log \sin x \sin y = \log C$$

$$\Rightarrow \sin x \cdot \sin y = C$$

Q. 58 The solution of $x\dfrac{dy}{dx} + y = e^x$ is

(a) $y = \dfrac{e^x}{x} + \dfrac{k}{x}$ (b) $y = xe^x + Cx$ (c) $y = xe^x + k$ (d) $x = \dfrac{e^y}{y} + \dfrac{k}{y}$

Sol. *(a)* Given that, $$x\frac{dy}{dx} + y = e^x$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{y}{x} = \frac{e^x}{x}$$

which is a linear differential equation.

$$\therefore \quad \text{IF} = e^{\int \frac{1}{x} dx} = e^{(\log x)} = x$$

The general solution is $y \cdot x = \int \left(\dfrac{d^x}{x} \cdot x \right) dx$

$$\Rightarrow \quad y \cdot x = \int e^x dx$$

$$\Rightarrow \quad y \cdot x = e^x + k$$

$$\Rightarrow \quad y = \frac{e^x}{x} + \frac{k}{x}$$

Q. 59 The differential equation of the family of curves $x^2 + y^2 - 2ay = 0$, where a is arbitrary constant, is

(a) $(x^2 - y^2)\dfrac{dy}{dx} = 2xy$ (b) $2(x^2 + y^2)\dfrac{dy}{dx} = xy$

(c) $2(x^2 - y^2)\dfrac{dy}{dx} = xy$ (d) $(x^2 + y^2)\dfrac{dy}{dx} = 2xy$

Sol. *(a)* Given equation of curve is

$$x^2 + y^2 - 2ay = 0$$

$$\Rightarrow \quad \frac{x^2 + y^2}{y} = 2a$$

On differentiating both sides w.r.t. x, we get

$$\frac{y\left(2x + 2y\dfrac{dy}{dx}\right) - (x^2 + y^2)\dfrac{dy}{dx}}{y^2} = 0$$

$$\Rightarrow \quad 2xy + 2y^2\frac{dy}{dx} - (x^2 + y^2)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad (2y^2 - x^2 - y^2)\frac{dy}{dx} = -2xy$$

$$\Rightarrow \quad (y^2 - x^2)\frac{dy}{dx} = -2xy$$

$$\Rightarrow \quad (x^2 - y^2)\frac{dy}{dx} = 2xy$$

Q. 60 The family $Y = Ax + A^3$ of curves will correspond to a differential equation of order

(a) 3 (b) 2

(c) 1 (d) not defined

Sol. *(c)* Given family of curves is $y = Ax + A^3$ ...(i)

$$\Rightarrow \quad \frac{dy}{dx} = A$$

Replacing A by $\frac{dy}{dx}$ in Eq. (i), we get

$$y = x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^3$$

$\therefore$ Order = 1

Q. 61 The general solution of $\frac{dy}{dx} = 2x\, e^{x^2 - y}$ is

(a) $e^{x^2 - y} = C$
(b) $e^{-y} + e^{x^2} = C$
(c) $e^y = e^{x^2} + C$
(d) $e^{x^2 + y} = C$

Sol. *(c)* Given that,

$$\frac{dy}{dx} = 2x\, e^{x^2 - y} = 2x\, e^{x^2} \cdot e^{-y}$$

$$\Rightarrow \quad e^y \frac{dy}{dx} = 2x\, e^{x^2}$$

$$\Rightarrow \quad e^y\, dy = 2x\, e^{x^2}\, dx$$

On integrating both sides, we get

$$\int e^y\, dy = 2\int x e^{x^2}\, dx$$

Put $x^2 = t$ in RHS integral, we get

$$2x\, dx = dt$$

$$\int e^y\, dy = \int e^t\, dt$$

$$\Rightarrow \quad e^y = e^t + C$$

$$\Rightarrow \quad e^y = e^{x^2} + C$$

Q. 62 The curve for which the slope of the tangent at any point is equal to the ratio of the abcissa to the ordinate of the point is

(a) an ellipse
(b) parabola
(c) circle
(d) rectangular hyperbola

Sol. *(d)* Slope of tangent to the curve $= \frac{dy}{dx}$

and ratio of abscissa to the ordinate $= \frac{x}{y}$

According to the question, $\frac{dy}{dx} = \frac{x}{y}$

$$y\,dy = x\,dx$$

On integrating both sides, we get

$$\frac{y^2}{2} = \frac{x^2}{2} + C$$

$$\Rightarrow \quad \frac{y^2}{2} - \frac{x^2}{2} = C \Rightarrow y^2 - x^2 = 2C$$

which is an equation of rectangular hyperbola.

Q. 63 The general solution of differential equation $\frac{dy}{dx} = e^{\frac{x^2}{2}} + xy$ is

(a) $y = Ce^{-x^2/2}$

(b) $y = C\,e^{x^2/2}$

(c) $y = (x + C)\,e^{x^2/2}$

(d) $y = (C - x)e^{x^2/2}$

Sol. *(c)* Given that, $\frac{dy}{dx} = e^{x^2/2} + xy$

$$\Rightarrow \quad \frac{dy}{dx} - xy = e^{x^2/2}$$

Here, $P = -x,\ Q = e^{x^2/2}$

$$\therefore \quad \text{IF} = e^{\int -x\,dx} = e^{-x^2/2}$$

The general solution is

$$y \cdot e^{-x^2/2} = \int e^{-x^2/2} - e^{x^2/2}\,dx + C$$

$$\Rightarrow \quad ye^{-x^2/2} = \int 1 d\,x + C$$

$$\Rightarrow \quad y \cdot e^{-x^2/2} = x + C$$

$$\Rightarrow \quad y = x\,e^{x^2/2} + C\,e^{+x^2/2}$$

$$\Rightarrow \quad y = (x + C)\,e^{x^2/2}$$

Q. 64 The solution of equation $(2y - 1)\,dx - (2x + 3)\,dy = 0$ is

(a) $\frac{2x - 1}{2y + 3} = k$

(b) $\frac{2y + 1}{2x - 3} = k$

(c) $\frac{2x + 3}{2y - 1} = k$

(d) $\frac{2x - 1}{2y - 1} = k$

Sol. *(c)* Given that, $(2y - 1)\,dx - (2x + 3)d\,y = 0$

$$\Rightarrow \quad (2y - 1)\,dx = (2x + 3)\,dy$$

$$\Rightarrow \quad \frac{dx}{2x + 3} = \frac{dy}{2y - 1}$$

On integrating both sides, we get

$$\frac{1}{2}\log(2x + 3) = \frac{1}{2}\log(2y - 1) + \log C$$

$$\Rightarrow \quad \frac{1}{2}[\log \cdot (2x + 3) - \log(2y - 1)] = \log C$$

$$\Rightarrow \quad \frac{1}{2}\log\left(\frac{2x + 3}{2y - 1}\right) = \log C$$

$$\Rightarrow \quad \left(\frac{2x + 3}{2y - 1}\right)^{1/2} = C$$

$$\Rightarrow \quad \frac{2x + 3}{2y - 1} = C^2$$

$$\Rightarrow \quad \frac{2x + 3}{2y - 1} = k, \text{ where } K = C^2$$

Q. 65 The differential equation for which $y = a \cos x + b \sin x$ is a solution, is

(a) $\frac{d^2y}{dx^2} + y = 0$ (b) $\frac{d^2y}{dx^2} - y = 0$

(c) $\frac{d^2y}{dx^2} + (a + b)\, y = 0$ (d) $\frac{d^2y}{dx^2} + (a - b)\, y = 0$

Sol. *(a)* Given that, $y = a \cos x + b \sin x$

On differentiating both sides w.r.t. x, we get

$$\frac{dy}{dx} = - a \sin x + b \cos x$$

Again, differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = - a \sin x + b \cos x$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = - y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} + y = 0$$

Q. 66 The solution of $\frac{dy}{dx} + y = e^{-x}$, $y(0) = 0$ is

(a) $y = e^{-x}(x - 1)$ (b) $y = xe^{x}$

(c) $y = xe^{-x} + 1$ (d) $y = xe^{-x}$

Sol. *(d)* Given that, $\frac{dy}{dx} + y = e^{-x}$

which is a linear differential equation.

Here, $P = 1$ and $Q = e^{-x}$

$$\text{IF} = e^{\int dx} = e^{x}$$

The general solution is

$$y \cdot e^{x} = \int e^{-x} \cdot e^{x}\, dx + C$$

$$\Rightarrow \quad ye^{x} = \int dx + C$$

$$\Rightarrow \quad ye^{x} = x + C \quad \text{...(i)}$$

When $x = 0$ and $y = 0$ then, $0 = 0 + C \Rightarrow C = 0$

Eq. (i) becomes $y \cdot e^{x} = x \Rightarrow y = x\,e^{-x}$

Q. 67 The order and degree of differential equation

$$\left(\frac{d^3y}{dx^3}\right)^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^4 = y^4 \text{ are}$$

(a) 1, 4 (b) 3, 4 (c) 2, 4 (d) 3, 2

Sol. *(d)* Given that, $\left(\frac{d^3y}{dx^3}\right)^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^4 = y^4$

$\therefore$ Order = 3

and degree = 2

Q. 68 The order and degree of differential equation $\left[1+\left(\frac{dy}{dx}\right)^2\right]=\frac{d^2y}{dx^2}$ are

(a) $2, \frac{3}{2}$ (b) 2, 3 (c) 2, 1 (d) 3, 4

Sol. *(c)* Given that, $\left[1+\left(\frac{dy}{dx}\right)^2\right]=\frac{d^2y}{dx^2}$

$\therefore$ Order $=2$ and degree $=1$

Q. 69 The differential equation of family of curves $y^2=4a(x+a)$ is

(a) $y^2=4\frac{dy}{dx}\left(x+\frac{dy}{dx}\right)$

(b) $2y\frac{dy}{dx}=4a$

(c) $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2=0$

(d) $2x\frac{dy}{dx}+y\left(\frac{dy}{dx}\right)^2-y=0$

Sol. *(d)* Given that, $y^2=4a(x+a)$ .. (i)

On differentiating both sides w.r.t. x, we get

$$2y\frac{dy}{dx}=4a \Rightarrow 2y\frac{dy}{dx}=4a$$

$$\Rightarrow y\frac{dy}{dx}=2a \Rightarrow a=\frac{1}{2}y\frac{dy}{dx} \quad ...(ii)$$

On putting the value of a from Eq. (ii) in Eq. (i), we get

$$y^2=2y\frac{dy}{dx}\left(x+\frac{1}{2}y\frac{dy}{dx}\right)$$

$$\Rightarrow y^2=2xy\frac{dy}{dx}+y^2\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow 2x\frac{dy}{dx}+y\left(\frac{dy}{dx}\right)^2-y=0$$

Q. 70 Which of the following is the general solution of $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$?

(a) $y=(Ax+B)e^x$

(b) $y=(Ax+B)e^{-x}$

(c) $y=Ae^x+Be^{-x}$

(d) $y=A\cos x+B\sin x$

Sol. *(a)* Given that, $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=0$

$$D^2y-2Dy+y=0,$$

where $D=\frac{d}{dx}$

$$(D^2-2D+1)y=0$$

The auxiliary equation is $m^2-2m+1=0$

$$(m-1)^2=0 \Rightarrow m=1, 1$$

Since, the roots are real and equal.

$\therefore$ $CF=(Ax+B)e^x \Rightarrow y=(Ax+B)e^x$

[since, if roots of Auxilliary equation are real and equal say (m), then $CF=(C_1x+C_2)e^{mx}$]

Q. 71 The general solution of $\frac{dy}{dx} + y \tan x = \sec x$ is

(a) $y \sec x = \tan x + C$ (b) $y \tan x = \sec x + C$
(c) $\tan x = y \tan x + C$ (d) $x \sec x = \tan y + C$

Sol. *(a)* Given differential equation is

$$\frac{dy}{dx} + y \tan x = \sec x$$

which is a linear differential equation

Here, $P = \tan x$, $Q = \sec x$,

$\therefore$ $\text{IF} = e^{\int \tan x \, dx} = e^{\log|\sec x|} = \sec x$

The general solution is

$$y \cdot \sec x = \int \sec x \cdot \sec x + C$$

$\Rightarrow$ $y \cdot \sec x = \int \sec^2 x \, dx + C$

$\Rightarrow$ $y \cdot \sec x = \tan x + C$

Q. 72 The solution of differential equation $\frac{dy}{dx} + \frac{y}{x} = \sin x$ is

(a) $x(y + \cos x) = \sin x + C$ (b) $x(y - \cos x) = \sin x + C$
(c) $xy \cos x = \sin x + C$ (d) $x(y + \cos x) = \cos x + C$

Sol. *(a)* Given differential equation is

$$\frac{dy}{dx} + y\frac{1}{x} = \sin x$$

which is linear differential equation.

Here, $P = \frac{1}{x}$ and $Q = \sin x$

$\therefore$ $\text{IF} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$

The general solution is

$$y \cdot x = \int x \cdot \sin x \, dx + C \quad \text{...(i)}$$

Take $I = \int x \sin x \, dx$

$$- x \cos x - \int - \cos x \, dx$$

$$= -x \cos x + \sin x$$

Put the value of I in Eq. (i), we get

$$xy = - x \cos x + \sin x + C$$

$\Rightarrow$ $x(y + \cos x) = \sin x + C$

Q. 73 The general solution of differential equation $(e^x + 1)\, y dy = (y + 1)\, e^x \, dx$ is

(a) $(y + 1) = k(e^x + 1)$ (b) $y + 1 = e^x + 1 + k$
(c) $y = \log\{k(y + 1)(e^x + 1)\}$ (d) $y = \log\left\{\frac{e^x + 1}{y + 1}\right\} + k$

Sol. *(c)* Given differential equation

$$(e^x + 1)\, y dy = (y + 1) e^x\, dx$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^x (1+y)}{(e^x + 1) y} \Rightarrow \frac{dx}{dy} = \frac{(e^x + 1)\, y}{e^x (1+y)}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{e^x y}{e^x (1+y)} + \frac{y}{e^x (1+y)}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{y}{1+y} + \frac{y}{(1+y) e^x}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{y}{1+y}\left(1 + \frac{1}{e^x}\right)$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{y}{1+y}\left(\frac{e^x + 1}{e^x}\right)$$

$$\Rightarrow \quad \left(\frac{y}{1+y}\right) dy = \left(\frac{e^x}{e^x + 1}\right) dx$$

On integrating both sides, we get

$$\int \frac{y}{1+y}\, dy = \int \frac{e^x}{1+e^x}\, dx$$

$$\Rightarrow \quad \int \frac{1+y-1}{1+y}\, dy = \int \frac{e^x}{1+e^x}\, dx$$

$$\Rightarrow \quad \int 1\, dy - \int \frac{1}{1+y}\, dy = \int \frac{e^x}{1+e^x}\, dx$$

$$\Rightarrow \quad y - \log|(1+y) = \log|(1+e^x) + \log k$$

$$\Rightarrow \quad y = \log(1+y) + \log(1+e^x) + \log(k)$$

$$\Rightarrow \quad y = \log\{k(1+y)(1+e^x)\}$$

Q. 74 The solution of differential equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ is

(a) $y = e^{x-y} - x^2 e^{-y} + C$

(b) $e^y - e^x = \frac{x^3}{3} + C$

(c) $e^x + e^y = \frac{x^3}{3} + C$

(d) $e^x - e^y = \frac{x^3}{3} + C$

Sol. *(b)* Given that,

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

$$\Rightarrow \quad \frac{dy}{dx} = e^x e^{-y} + x^2 e^{-y}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{e^x + x^2}{e^y}$$

$$\Rightarrow \quad e^y\, dy = (e^x + x^2)\, dx$$

On integrating both sides, we get

$$\int e^y\, dy = \int (e^x + x^2)\, dx$$

$$\Rightarrow \quad e^y = e^x + \frac{x^3}{3} + C$$

$$\Rightarrow \quad e^y - e^x = \frac{x^3}{3} + C$$

Q. 75 The solution of differential equation $\frac{dy}{dx}+\frac{2xy}{1+x^2}=\frac{1}{(1+x^2)^2}$ is

(a) $y(1+x^2)=C+\tan^{-1}x$

(b) $\frac{y}{1+x^2}=C+\tan^{-1}x$

(c) $y\log(1+x^2)=C+\tan^{-1}x$

(d) $y(1+x^2)=C+\sin^{-1}x$

Sol. *(a)* Given that, $\frac{dy}{dx}+\frac{2xy}{1+x^2}=\frac{1}{(1+x^2)^2}$

Here, $P=\frac{2x}{1+x^2}$ and $Q=\frac{1}{(1+x^2)^2}$

which is a linear differential equation.

$\therefore$ $\text{IF}=e^{\int\frac{2x}{1+x^2}dx}$

Put $1+x^2=t\Rightarrow 2x\,dx=dt$

$\therefore$ $\text{IF}=e^{\int\frac{dt}{t}}=e^{\log t}=e^{\log(1+x^2)}=1+x^2$

The general solution is

$$y\cdot(1+x^2)=\int(1+x^2)\frac{1}{(1+x^2)^2}+C$$

$$\Rightarrow\quad y(1+x^2)=\int\frac{1}{1+x^2}dx+C$$

$$\Rightarrow\quad y(1+x^2)=\tan^{-1}x+C$$

Fillers

Q. 76 (i) The degree of the differential equation $\frac{d^2y}{dx^2}+e^{dy/dx}=0$ is ....

(ii) The degree of the differential equation $\sqrt{1+\left(\frac{dy}{dx}\right)^2}=x$ is ....

(iii) The number of arbitrary constants in the general solution of a differential equation of order three is ....

(iv) $\frac{dy}{dx}+\frac{y}{x\log x}=\frac{1}{x}$ is an equation of the type ....

(v) General solution of the differential equation of the type is given by ....

(vi) The solution of the differential equation $\frac{xdy}{dx}+2y=x^2$ is ....

(vii) The solution of $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$ is ....

(viii) The solution of the differential equation $ydx + (x + xy)\, dy = 0$ is ....

(ix) General solution of $\frac{dy}{dx} + y = \sin x$ is ....

(x) The solution of differential equation $\cot y\, dx = xdy$ is ....

(xi) The integrating factor of $\frac{dy}{dx} + y = \frac{1+y}{x}$ is ....

Sol. **(i)** Given differential equation is

$$\frac{d^2y}{dx^2} + e^{\frac{dy}{dx}} = 0$$

Degree of this equation is not defined.

(ii) Given differential equation is $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = x$

So, degree of this equation is two.

(iii) There are three arbitrary constants.

(iv) Given differential equation is $\frac{dy}{dx} + \frac{y}{x \log x} = \frac{1}{x}$

The equation is of the type $\frac{dy}{dx} + Py = Q$

(v) Given differential equation is

$$\frac{dx}{dy} + P_1 x = Q_1$$

The general solution is

$$x \cdot \text{IF} = \int Q\,(\text{IF})\, dy + C \;\; i.e., x\, e^{\int P dy} = \int Q\, \{e^{\int P dy}\}\, dy + C$$

(vi) Given differential equation is

$$x\frac{dy}{dx} + 2y = x^2 \Rightarrow \frac{dy}{dx} + \frac{2y}{x} = x$$

This equation of the form $\frac{dy}{dx} + Py = Q$.

$$\therefore \qquad \text{IF} = e^{\int \frac{2}{x} dx} = e^{2 \log x} = x^2$$

The general solution is

$$yx^2 = \int x \cdot x^2\, dx + C$$

$$\Rightarrow \qquad yx^2 = \frac{x^4}{4} + C$$

$$\Rightarrow \qquad y = \frac{x^2}{4} + Cx^{-2}$$

(vii) Given differential equation is

$$(1 + x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0$$

$$\Rightarrow \qquad \frac{dy}{dx} + \frac{2xy}{1 + x^2} - \frac{4x^2}{1 + x^2} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{4x^2}{1+x^2}$$

$$\therefore \quad \text{IF} = e^{\int \frac{2x}{1+x^2} dx}$$

Put $\quad 1 + x^2 = t \Rightarrow 2x dx = dt$

$$\therefore \quad \text{IF} = e^{\int \frac{dt}{t}} = e^{\log t} = e^{\log (1+x^2)} = 1 + x^2$$

The general solution is

$$y \cdot (1+x^2) = \int (1+x^2) \frac{.4x^2}{(1+x^2)} dx + C$$

$$\Rightarrow \quad (1+x^2)\, y = \int 4x^2 \, dx + C$$

$$\Rightarrow \quad (1+x^2) y = 4 \frac{x^3}{3} + C$$

$$\Rightarrow \quad y = \frac{4x^3}{3(1+x^2)} + C\,(1+x^2)^{-1}$$

(viii) Given differential equation is

$$\Rightarrow \quad ydx + (x + xy)\, dy = 0$$

$$\Rightarrow \quad ydx + x\,(1+y)\, d\, y = 0$$

$$\Rightarrow \quad \frac{dx}{-x} = \left(\frac{1+y}{y}\right) dy$$

$$\Rightarrow \quad \int \frac{1}{x} dx = -\int \left(\frac{1}{y} + 1\right) dy \qquad \text{[on integrating]}$$

$$\Rightarrow \quad \log (x) = -\log (y) - y + \log A$$

$$\log (x) + \log (y) + y = \log A$$

$$\log (xy) + y = \log A$$

$$\Rightarrow \quad \log xy + \log e^y = \log A$$

$$\Rightarrow \quad xy\, e^y = A$$

$$\Rightarrow \quad xy = Ae^{-y}$$

(ix) Given differential equation is

$$\frac{dy}{dx} + y = \sin x$$

$$\text{IF} = \int e^{1dx} = e^x$$

The general solution is

$$y \cdot e^x = \int e^x \sin x \, dx + C \qquad \text{...(i)}$$

Let $\quad I = \int e^x \sin x \, dx$

$$I = \sin x \, e^x - \int \cos x \, e^x \, dx$$

$$= \sin x \, e^x - \cos x \, e^x + \int (-\sin x) e^x \, dx$$

$$2I = e^x (\sin x - \cos x)$$

$$I = \frac{1}{2} e^x (\sin x - \cos x)$$

From Eq. (i),

$$y \cdot e^x = \frac{x}{2}(\sin x - \cos x) + C$$

$$\Rightarrow \quad y = \frac{1}{2}(\sin x - \cos x) + C \cdot e^{-x}$$

(x) Given differential equation is

$$\cot y\, dx = x dy$$

$$\Rightarrow \quad \frac{1}{x} dx = \tan y\, dy$$

On integrating both sides, we get

$$\Rightarrow \quad \int \frac{1}{x} dx = \int \tan y\, d\, y$$

$$\Rightarrow \quad \log(x) = \log(\sec y) + \log C$$

$$\Rightarrow \quad \log\left(\frac{x}{\sec y}\right) = \log C$$

$$\Rightarrow \quad \frac{x}{\sec y} = C$$

$$\Rightarrow \quad x = C \sec y$$

(xi) Given differential equation is

$$\frac{dy}{dx} + y = \frac{1+y}{x}$$

$$\frac{dy}{dx} + y = \frac{1}{x} + \frac{y}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} + y\left(1 - \frac{1}{x}\right) = \frac{1}{x}$$

$$\therefore \quad \text{IF} = e^{\int\left(1 - \frac{1}{x}\right)dx}$$

$$= e^{x - \log x}$$

$$= e^x \cdot e^{-\log x} = \frac{e^x}{x}$$

True/False

Q. 77 State True or False for the following

(i) Integrating factor of the differential of the form $\frac{dx}{dy} + P_1 x = Q_1$ is given by $e^{\int P_1 dy}$.

(ii) Solution of the differential equation of the type $\frac{dx}{dy} + P_1 x = Q_1$ is given by $x \cdot \text{IF} = \int (\text{IF}) \times Q_1\, dy$.

(iii) Correct substitution for the solution of the differential equation of the type $\frac{dy}{dx} = f(x, y)$, where $f(x, y)$ is a homogeneous function of zero degree is $y = vx$.

(iv) Correct substitution for the solution of the differential equation of the type $\frac{dy}{dx} = g(x, y)$, where $g(x, y)$ is a homogeneous function of the degree zero is $x = vy$.

(v) Number of arbitrary constants in the particular solution of a differential equation of order two is two.

(vi) The differential equation representing the family of circles $x^2 + (y - a)^2 = a^2$ will be of order two.

(vii) The solution of $\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3}$ is $y^{2/3} - x^{2/3} = c$

(viii) Differential equation representing the family of curves $y = e^x(A \cos x + B \sin x)$ is $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$.

(ix) The solution of the differential equation $\frac{dy}{dx} = \frac{x + 2y}{x}$ is $x + y = kx^2$.

(x) Solution of $\frac{xdy}{dx} = y + x \tan \frac{y}{x}$ is $\sin\left(\frac{y}{x}\right) = cx$

(xi) The differential equation of all non horizontal lines in a plane is $\frac{d^2x}{dy^2} = 0$.

Sol. **(i)** ***True***

Given differential equation,

$$\frac{dx}{dy} + P_1 x = Q_1$$

$\therefore$ $$\text{IF} = e^{\int p_1 dy}$$

(ii) ***True***

(iii) ***True***

(iv) ***True***

(v) ***False***

There is no arbitrary constant in the particular solution of a differential equation.

(vi) ***False***

We know that, order of the differential equation = number of arbitrary constant

Here, number of arbitrary constant = 1.

So order is one.

(vii) ***True***

Given differential equation, $$\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y^{1/3}}{x^{1/3}}$$

$$\Rightarrow \quad y^{-1/3}\, dy = x^{-1/3}\, dx$$

On integrating both sides, we get

$$\int y^{-1/3}\, dy = \int x^{-1/3} dx$$

$$\Rightarrow \quad \frac{y^{-1/3+1}}{\frac{-1}{3}+1} = \frac{x^{-1/3+1}}{\frac{-1}{3}+1} + C'$$

$$\Rightarrow \quad \frac{3}{2} y^{2/3} = \frac{3}{2} x^{2/3} + C'$$

$$\Rightarrow \quad y^{2/3} - x^{2/3} = C' \qquad \left[\text{where, } \frac{2}{3} C' = C\right]$$

(viii) ***True***

Given that, $$y = e^x (A\cos x + B\sin x)$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = e^x(-A\sin x + B\cos x) + e^x(A\cos x + B\sin x)$$

$$\Rightarrow \quad \frac{dy}{dx} - y = e^x(-A\sin x + B\cos x)$$

Again differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} = e^x(-A\cos x - B\sin x) + e^x(-A\sin x + B\cos x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - \frac{dy}{dx} + y = \frac{dy}{dx} - y$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

(ix) ***True***

Given that, $$\frac{dy}{dx} = \frac{x + 2y}{x} \Rightarrow \frac{dy}{dx} = 1 + \frac{2}{x}.y$$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{2}{x}y = 1$$

$$\text{IF} = e^{\int \frac{-2}{x} dx} = e^{-2\log x} = x^{-2}$$

The differential solution,

$$y \cdot x^{-2} = \int x^{-2} \cdot 1 dx + k$$

$$\Rightarrow \quad \frac{y}{x^2} = \frac{x^{-2+1}}{-2+1} + k$$

$$\Rightarrow \quad \frac{y}{x^2} = \frac{-1}{x} + k$$

$$\Rightarrow \quad y = -x + kx^2$$

$$\Rightarrow \quad x + y = kx^2$$

(x) ***True***

Given differential equation,

$$\frac{xdy}{dx} = y + x \tan\left(\frac{y}{x}\right)$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right). \quad \ldots(i)$$

Put $\quad \frac{y}{x} = v$ *i.e.*, $y = vx$

$$\Rightarrow \quad \frac{dy}{dx} = v + \frac{xdv}{dx}$$

On substituting these values in Eq. (i), we get

$$\frac{xdv}{dx} + v = v + \tan v$$

$$\Rightarrow \quad \frac{dx}{x} = \frac{dv}{\tan v}$$

On integrating both sides, we get

$$\int \frac{1}{x}\,dx = \int \frac{1}{\tan v}\,dx$$

$$\Rightarrow \quad \log(x) = \log(\sin v) + \log C'$$

$$\Rightarrow \quad \log\left(\frac{x}{\sin v}\right) = \log C'$$

$$\Rightarrow \quad \frac{x}{\sin v} = C'$$

$$\Rightarrow \quad \sin v = Cx \quad \left[\text{where, } C = \frac{1}{C'}\right]$$

$$\Rightarrow \quad \sin\frac{y}{x} = Cx$$

(xi) ***True***

Let any non-horizontal line in a plane is given by

$$y = mx + c$$

On differentiating w.r.t. x, we get

$$\frac{dy}{dx} = m$$

Again, differentiating w.r.t. x, we get

$$\frac{d^2y}{dx^2} = 0$$